Some New Families of Edge Odd Graceful Graphs

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Abstract-A (p, q) connected graph is edge-odd graceful graph if their exists an injective map f:E(G)→{1,3,…,2q-1} so that the induced map f_+:V(G)→{0,1,2,…,(2k-1)} defined by f_+ (x)≡∑{f(x,y)/xy∈E } (mod 2k) where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. When the graph admits the edge odd graceful labeling, the graph is called edge-odd graceful graph. In this article, the edge – odd gracefulness of direct sum graph and square of the Graph G is obtained.

Keywords: Graph labeling, Graceful labeling, Edge odd graceful labeling, direct sum graph, square of the graph G

I Introduction:

Definition 1.1:- If the vertices are assigned values subject to certain conditions then it is known as graph labeling. Graph labelings is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and graph decomposition problems. For an complete survey on graph labeling and related results we refer Gallian J.A. [1]

Most of the graph labeling problems found their origin with that of graceful labeling introduced by Rosa. A [2]

Definition 1.2:- A function f is called graceful labeling of a graph if f : V(G) → {0, 1, 2 … q} is injective function and the induced function f* $E(G) \longrightarrow \{1, 2, \dots, q\}$ defined as

f*(e = uv) = ∣ f(x) – f (y) ∣ is bijective. A graph which admits graceful labeling is called a graceful graph.

Definition 1.3:- A (p, q) connected graph is edge - odd graceful graph if there exists an injective map: E(G) → {1, 3, …, 2q-1} so that induced map f_+:V(G)→{0,1,2,…,(2k-1)}defined by $f_+(x) \equiv \sum\{f(x, y)/ xy \in E\}$(mod2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd.

Definition 1.4:- $k_1 \oplus 6p_n$ is a direct sum graph, connected from 5 copies of P_n (whose vertices u_1, u_2, …u_{n-1}; u_1, v_2, v_3, …, v_{n-1}, u_n is first copy of P_n; u_1, w_2, …,w_{n-1}, u_n is second copy of P_n ; u_1, s_2, … s_{n-1}, $u_n$is third copy of P_n; u_1, x_1, x_2, …, x_{n-1}, u_n is fourth copy of P_n; u_1, y_2, y_3…, y_{n-1}, u_n is fifth copy of P_n ; u_1, z_2, …, z_{n-1}, u_n is sixth copy of P_n) and a null vertex t(n_1) whose adjacency edges other than existing edges are tu_i (for every $1 \le i \le n$) ; $u_i v_i$, $2 \le i \le n-1$; v_j

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w_j, $2 \le j \le$ n-1; w_k s_k, $2 \le k \le$ n-1; s_l x_l, $2 \le l \le$ n-1; x_m y_m, $2 \le m \le$ n-1; y_n z_n, $2 \le n \le$ n-1))

Definition 1.5. For a simple connected graph G the *square of graph* G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1or 2 apart in G.

Main result:

Theorem 2.1: $k_1 \oplus 6p_n$ is edge odd graceful for all n.

Proof:- Consider six copies of $k_1 \oplus 6p_n$ .Vertices and edges are defined in the definition 1.4. Note that $|V(k_1 \oplus 6p_n)|$= 7 n-11 and $|E(k_1 \oplus 6p_n)|$= 14n-19.

Define edge labeling $f: E(k_1 \oplus 6p_n \rightarrow \{1,3,\dots,2q-1\}$ as follows.

Case (i): $n \equiv 0\ (mod 4)$

$f(e_i) = 2n - 1$

$f(e_i) = f(e_{13n-8}) + 2(i - 1),\ 2 \le i \le n$

$f(e_i) = 2i - 1,\ n+1 \le i \le 13n - 18$

$f(e_{13n-18+i}) = 2i - 1,\ 1 \le i \le n - 3$

$f(e_{14n-20}) = 2n - 3,\ f(e_{14n-19}) = 2n - 5$

Case (ii): $n \equiv 1\ (mod 4)$

$f(e_i) = 2i - 1,\ n+1 \le i \le 13n - 18$

$f(e_i) = f(e_{13n-18}) + 2(i - 1),\ 3 \le i \le n$

$f(e_i) = f(e_{13n-18}) + 2i,\ i = 1$

$f(e_{13n-18+i}) = 2i - 1,\ 1 \le i \le n - 1$

$f(e_2) = 2n - 1$

Case (iii): $n \equiv 2\ (mod 4)$

$f(e_i) = 2i - 1,\ n+1 \le i \le 13n - 18$

$f(e_i) = f(e_{13n-18}) + 2(i - 1),\ 2 \le i \le n$

$f(e_1) = 2n - 1$

$f(e_{13n-18+i}) = 2i - 1,\ 1 \le i \le n - 3$

$f(e_{14n-20}) = 2n - 3,\ f(e_{14n-19}) = 2n - 1$

Case (iv): $n \equiv 3\ (mod 4)$

$f(e_i) = 2i - 1,\ n+1 \le i \le 13n - 18$

$f(e_{13n-18+i}) = 2i - 1,\ 1 \le i \le n - 1$

$f(e_i) = f(e_{13n-18}) + 2(i - 1),\ 3 \le i \le n$

$f(e_2) = 2n - 1,\ \ f(e_i) = f(e_{13n-18}) + 2i,\ i=1$

The above defined edge labeling function will induce the bijective vertex labeling function $f_+: V(G) \rightarrow \{0,1,2,\dots,(2k-1)\}$ such that $f_+(x) \equiv \sum\{f(x, y) / xy \in E\} (mod 2k)$, where k = max{p, q}.

Thus we proved that $k_1 \oplus 6p_n$ admits edge-odd graceful labeling.

Example 1: Edge odd graceful labeling of $k_1 \oplus 6p_3$ is shown in figure 1.

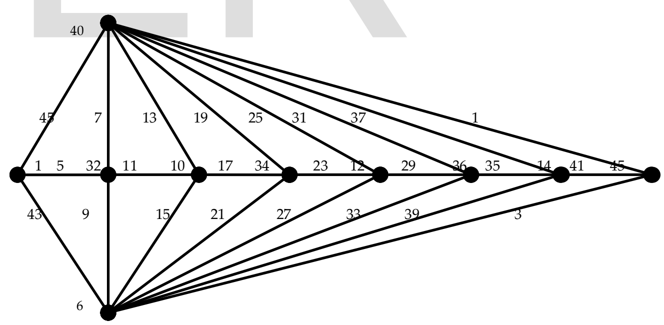

Figure 1- Edge odd gracefulness of $k_1 \oplus 6p_3$

Theorem 2.2: $B^2_{n,n}$ is a graceful graph.

Proof: Consider $B_{n,n}$ with the vertex set {u, v, u_i, v_i, $1 \le i \le n$} where u_i, v_i are the pendant vertices. Let G be the graph $B^2_{n,n}$ then |V (G)| = 2n + 2 and |E(G)| = 4n + 1.

Define edge labeling $f: E(B^2) \rightarrow \{1,3,\dots,2q-1\}$ as follows. $f(e_i) = 2i - 1,\ 3 \le i \le (4n+1)$, $f(e_1) = 3$, $f(e_2) = 1$

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The above defined edge labeling function will induce the bijective vertex labeling function $f_+:V(G)\rightarrow\{0,1,2,\ldots,(2k-1)\}$ such that

$f_+(x) \equiv \sum \{f(x, y) / xy \in E\}$ (mod2k),

where k = max{p,q}.Thus we proved that $B^2_{n,n}$ admits edge-odd graceful labeling.

Example 2: Edge odd graceful labeling of $B^2_{3,3}$ is shown in figure 2.

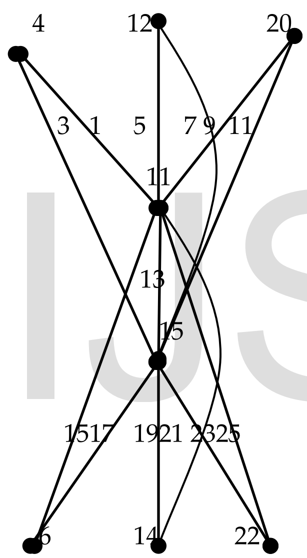

Figure 2- Edge odd gracefulness of $B^2_{3,3}$

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